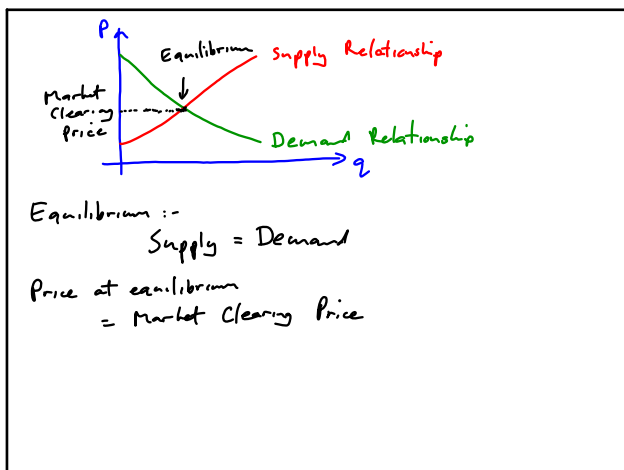
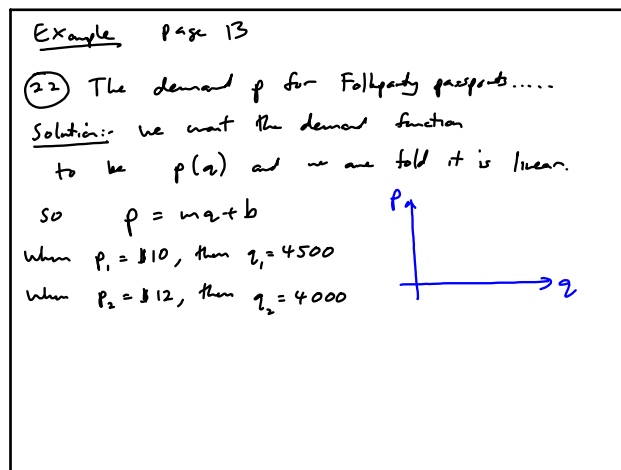
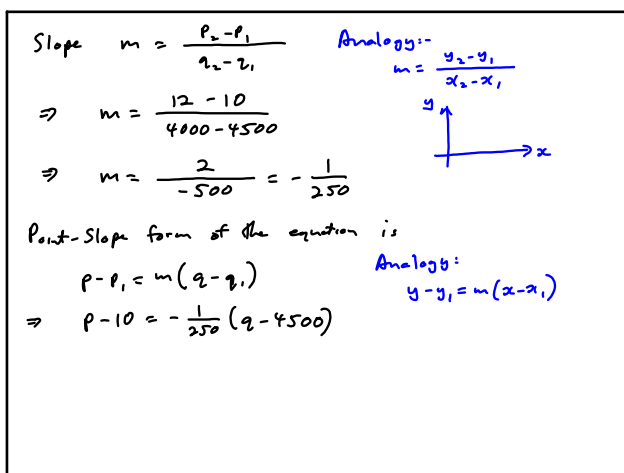




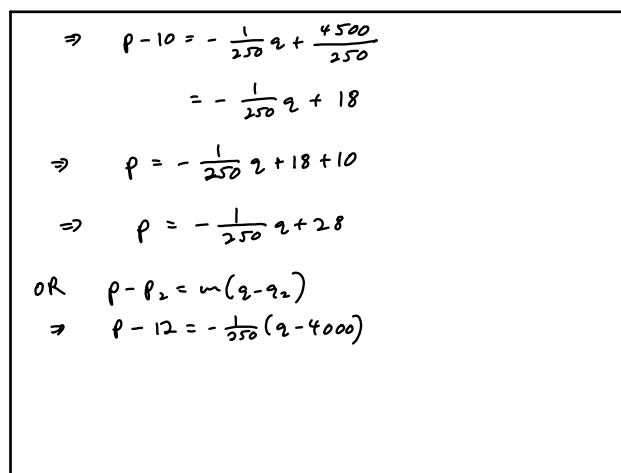
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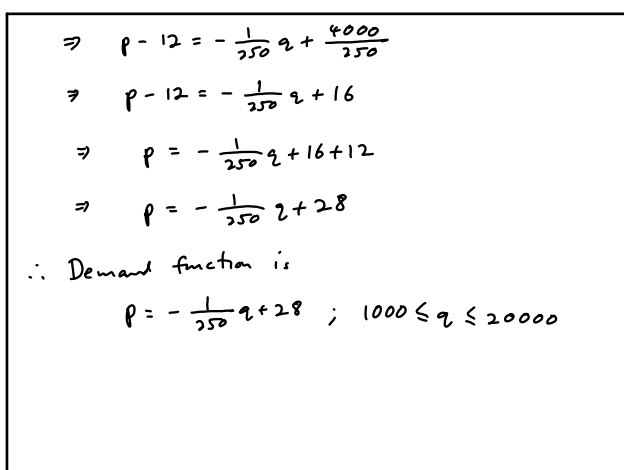
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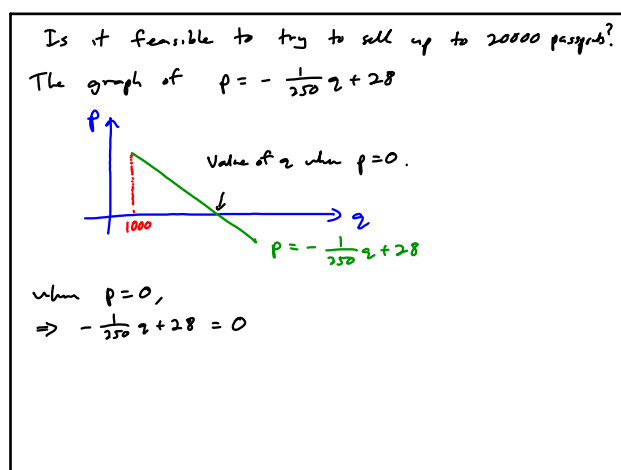
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May 11-9:00 AM



May 11-9:02 AM

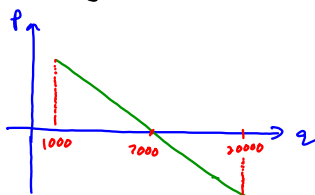


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$$\Rightarrow \frac{1}{250} q = 28$$

$$\Rightarrow q = (250)(28) = 7000$$

so the reality is



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No, it is not feasible to try to sell up to 20000 passports.

(23) The linear supply function.....

Solution:- We are told the supply function is linear

$$p = mq + b$$

When $p_1 = \$10$, then $q_1 = 3500$

When $p \uparrow$ by $\$2$, then $q \uparrow$ by 1000



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so slope, $m = \frac{\text{rise}}{\text{run}}$

$$\Rightarrow m = \frac{2}{1000} = \frac{1}{500}$$

The point-slope form of the line is

$$p - p_1 = m(q - q_1)$$

$$\Rightarrow p - 10 = \frac{1}{500}(q - 3500)$$

$$\Rightarrow p - 10 = \frac{1}{500}q - \frac{3500}{500}$$

$$\Rightarrow p - 10 = \frac{1}{500}q - 7$$

$$\Rightarrow p = \frac{1}{500}q - 7 + 10$$

$$\Rightarrow p = \frac{1}{500}q + 3 ; 0 \leq q \leq 20000$$

Note:- We saw in the earlier question that in actual fact, it is not advisable to make up to 20000 passports. It is advisable to print just up to 7000 passports.

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(24) Using the supply and demand.....

Solution:- Equilibrium occurs when

supply = demand

$$\Rightarrow \frac{1}{500}q + 3 = -\frac{1}{250}q + 28$$

$$\Rightarrow \frac{1}{500}q + \frac{1}{250}q = 28 - 3$$

$$\Rightarrow \frac{3}{500}q = 25$$

$$\Rightarrow 3q = (25)(500) = 12500$$

$$\Rightarrow q = \frac{12500}{3}$$

$$\Rightarrow q = 4166 \frac{2}{3}$$

Market clearing price can be found using either the supply function or the demand function.

Demand function:- $p = -\frac{1}{250}q + 28$

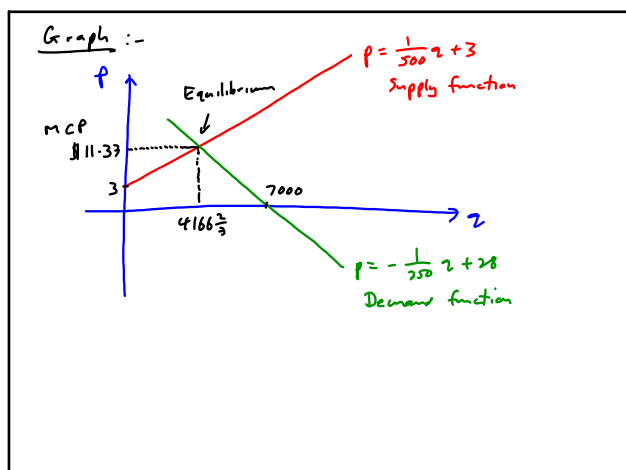
$$\text{so MCP} = -\frac{1}{250}(4166 \frac{2}{3}) + 28 \approx \$11.33$$

OR Supply function:- $p = \frac{1}{500}q + 3$

$$\text{so MCP} = \frac{1}{500}(4166 \frac{2}{3}) + 3 \approx \$11.33$$

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1.3 Quadratic Functions

All quadratic functions can be written as

$$y = ax^2 + bx + c ; a \neq 0$$

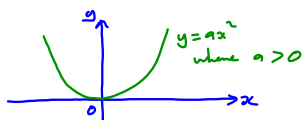
Let's begin our discussion with the simplest quadratic function

$$y = ax^2 ; a \neq 0$$

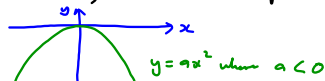
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Two possibilities:-

- ① If $a > 0$, then the graph is a "smiley".



- ② If $a < 0$, then the graph is a "frowny".



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This is true for all quadratics

$$y = ax^2 + bx + c$$

Whether a quadratic graph is a "smiley" or a "frowny" depends only on the sign of the coefficient a .If $a = +ve$, graph is "smiley"If $a = -ve$, graph is "frowny"All quadratics draw parabolas.

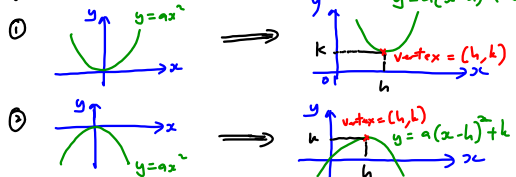
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If we shift the graph of $y = ax^2$

- ① horizontally by h units

and ② vertically by k units

we obtain either:-



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All quadratics $y = ax^2 + bx + c$

can be rewritten in the vertex form

$$y = a(x-h)^2 + k$$

where (h, k) is where the vertex is located.

We use the "Completing the Square" technique

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Completing the Square :-

We start with

$$y = ax^2 + bx + c$$

- ① Factor out the coefficient of x^2 from the first two terms

$$y = ax^2 + bx + c$$

$$\Rightarrow y = a\left(x^2 + \frac{b}{a}x\right) + c$$

- ② Take the coefficient of x within the brackets, halve it and square the result

$$\left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

- ③ Add and subtract the result from step ② within the brackets of the result from step ①.

$$y = ax^2 + bx + c$$

$$\Rightarrow y = a\left(x^2 + \frac{b}{a}x\right) + c$$

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$$\Rightarrow y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$

And we simplify:-

$$\Rightarrow y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + a\left(-\frac{b^2}{4a^2}\right) + c$$

$$\Rightarrow y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$\Rightarrow y = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + c - \frac{b^2}{4a}$$

Compare with $y = a(x-h)^2 + k$

We see that

$$h = -\frac{b}{2a} \quad \text{and} \quad k = c - \frac{b^2}{4a}$$

We can simplify k further,

$$\begin{aligned} k &= c - \frac{b^2}{4a} \\ &= c - a\left(\frac{b^2}{4a^2}\right) \\ &= c - a\left(-\frac{b}{2a}\right)^2 \end{aligned}$$

$$\Rightarrow k = c - ah^2$$

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Summary :-Given a quadratic $y = ax^2 + bx + c$

we can rewrite it as

$$y = a(x-h)^2 + k$$

where its vertex is located

at $(h, k) = \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

Example page 19

- ② Sketch $p = a^2 + 12a - 20$

Solution :-

Vertex is located at

$$(h, k) = \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$$

We have $a = 1$, $b = 12$ and $c = -20$

$$h = -\frac{b}{2a} = -\frac{12}{2(1)} = -6$$

$$\text{and } k = c - ah^2 = -20 - (1)(-6)^2 = -20 - 36 = -56$$

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② To find the q -intercept, we plug in $p=0$ and solve for q

$$\text{so } q^2 + 12q - 20 = 0$$

Review:-

To solve $ax^2 + bx + c = 0$

① Rewrite as $a(x-r_1)(x-r_2) = 0$ if possible

$$\Rightarrow x - r_1 = 0 \text{ or } x - r_2 = 0$$

$$\Rightarrow x = r_1 \text{ or } x = r_2$$

② Use the quadratic formula:-

$$ax^2 + bx + c = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Leftarrow \text{Quadratic Formula}$$

Back to our problem:-

$$q^2 + 12q - 20 = 0$$

$$\Rightarrow q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{12^2 - 4(1)(-20)}}{2(1)}$$

$$= \frac{-12 \pm \sqrt{224}}{2} \approx \frac{-12 \pm 14.9}{2}$$

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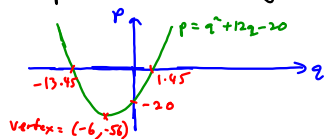
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$$\Rightarrow q = \frac{-12 - 14.9}{2} \text{ or } q = \frac{-12 + 14.9}{2}$$

$$\Rightarrow q = \frac{-26.9}{2} \text{ or } q = \frac{2.9}{2}$$

$$\Rightarrow q = -13.45 \text{ or } q = 1.45$$

As the parabola is a "smiley" since $a > 0$



③ The McDonalds -

we want the revenue function,

$$R = pq$$

Given that $p = -0.04q + 13$

$$\text{so } R = (-0.04q + 13)(q)$$

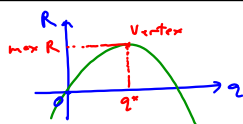
$$\Rightarrow R = -0.04q^2 + 13q ; 0 \leq q \leq 325$$

We want to maximize revenue

R is a "frowny" quadratic

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Vertex is when the max revenue is for $R(q)$

$$R = -0.04q^2 + 13q$$

$$\text{vertex } (h, k) = \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$$

Here $a = -0.04$, $b = 13$ and $c = 0$

$$\text{so } h = q^* = -\frac{b}{2a} = -\frac{13}{2(-0.04)} = 162.5$$

We need to sell $162\frac{1}{2}$ burgers to maximize revenue.

$\therefore$ max. revenue is

$$R = -0.04q^2 + 13q$$

$$= -0.04(162.5)^2 + 13(162.5)$$

$$= \$1,056.25$$

Selling price at $162\frac{1}{2}$ burgers

$$\text{is } p = -0.04q + 13$$

$$\Rightarrow p = -0.04(162.5) + 13 = \$6.50$$

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